# Comparison of Stratum-Specific Likelihood Ratios and 2x2 Approaches

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Epi 550

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### Dismissal of Sensitivity and Specificity

"Readers who have followed the discussion [about likelihood ratios] to this point will understand the essentials of interpretation of diagnostic tests and can stop here.

"They should consider the next section, which deals with sensitivity and specificity, optional.

"We include it largely because clinicians will encounter studies that present their results in terms of sensitivity and specificity and may wish to understand this alternative framework for summarizing properties of diagnostic tests."

Jaeschke, et al. JAMA. 1994;271:703-707



### 2 Types of Decision Making

- Previously introduced "one (test) and done," 1 of (at least) 2 types of decision making
  - One and done decision making characterized by need/desire to make treatment decision quickly without a large number of opportunities to collect additional data
- Second type of decision making referred to as "continuous updating"
  - Continuous updating characterized by performing a number of different tests and regularly updating probability of disease based on each test's results
- Today focus on strengths and weakness of 2x2 and SSLR approaches for these 2 decision making paradigms



# Relative Merits of Optimal 2x2 Tables and SSLR for One (Test) and Done Decision Making

- In One and Done decision making, use of OPTIMAL 2x2 table and SSLR yield identical treatment decisions, because for optimal 2x2 table:
  - All strata with SSLR yielding post-test probabilities above treatment threshold (p\*) are classified as positive test results;
  - All strata with SSLR yielding post-test probabilities below treatment threshold are classified as negative test results

NOTE: ALL SSLR YIELDING POST-TEST PROBABILITIES ABOVE P\*, NOT ALL 2x2 TABLES



	SSLR, Not 2x2 Tables!!!							
Cut-off	Stratum	Bact	No Bact	LR+	SSLR			
<u>&gt;</u> 25	<u>&gt;</u> 25	6	26	7.624	7.624			
<u>&gt;</u> 20	<u>&gt;</u> 20, <25	4	43	4.788	3.073			
<u>&gt;</u> 15	<u>&gt;</u> 15, <20	7	129	2.837	1.793			
<u>&gt;</u> 10	<u>&gt;</u> 10, <15	7	292	1.618	0.792			
<u>&gt;</u> 0 *	<u>&gt;</u> 0, <10 *	2	369		0.179			
Total	Total	26	859					
<u>&gt;</u> 25 incl	e that p=0.2, uded as a po _R of stratum	sitive to	est becaus	e 1) LR·	+ of T+ <u>&gt;</u> 2			
• And 2) b	ooth yield a p	ost-tes	t probabilit	y > 0.5 (	p*)			
	<b>∂</b> .0 < 99.0 = (	8.0 + (428	9.7 * 2.0) )\(\$2	:9.7 * 2.0)	0000			

Cut off			2x2 Table		
Cut-off	Stratum	Bact	No Bact	LR+	SSLR
<u>&gt;</u> 25	<u>&gt;</u> 25	6	26	7.624	7.624
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Total	Total	26	859		
	=0.2, ∆O <sub>D+</sub> = also yields p s <sup>.</sup> 0 < ⊭s <sup>.</sup> 0 = (	ost-test		/ > 0.5	).5, LR+ (
	s that mean sidered a pos			should b	be



# Should <a>20, <25 Be Considered a Positive Test?</a>

- NO!
- While LR+ (4.788) for T+ ≥20 raises post-test probability above 0.5…

(0.2\*4.788) / ((0.2\*4.788) + 0.8) = .5438 > 0.5 • SSLR (3.073) for ≥20, <25 does not

(0.2\*3.073) / ((0.2\*3.073) + 0.8) = .4345 < 0.5

- 2x2 table with ≥20 cut-off yields a post-test probability above 0.5 because information it borrows from ≥25 stratum is highly predictive
- NOT because information from <u>></u>20/<25 stratum is highly predictive</li>



Reiteration

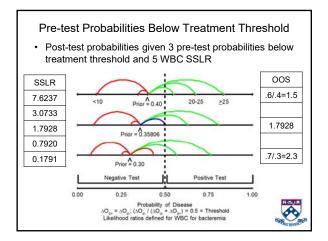
Method for identifying optimal 2x2 table ensures classification of strata with SSLR yielding post-test probabilities above treatment threshold as positive tests

Also ensures classification of strata with SSLR yielding post-test probabilities below treatment threshold as negative tests



	tration: In One a d SSLR Approa		0.
• WBC S	SLR for Bacterem	а	
	Strata	SSLR	
	<u>&gt;</u> 25	7.6237	
	<u>&gt;</u> 20, <25	3.0733	
	<u>&gt;</u> 15, <20	1.7928	
	<u>&gt;</u> 10, <15	0.7920	
	<u>&gt;</u> 0, <10	0.1791	
• Assum	e that $\Delta O_{D+} = \Delta O_{D-}$	$\rightarrow$ p* = 0.50	A



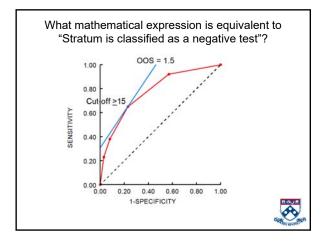




Proof: For one and Done Decision Making, 2x2 and SSLR Approaches Yield Same Treatment Decision

 Is it possible for 1) a stratum to be classified as a negative test if 2) post-test probability resulting from its stratum specific likelihood ratio is above treatment threshold?







What mathematical expression is equivalent to "Post-test probability is above treatment threshold"?

2) Post-test Probability Above Treatment Threshold  
1) Post-Test > P\*  
2) Post-Test = 
$$\frac{SSLR_i p}{(SSLR_i p) + (1-p)}$$
  
3) P\* =  $\frac{\Delta O_d}{\Delta O_{d-} + \Delta O_{d+}}$   
4)  $\frac{SSLR_i p}{(SSLR_i p) + (1-p)} > \frac{\Delta O_{d-}}{\Delta O_{d-} + \Delta O_{d+}}$ 

$$\begin{aligned} & \text{Rearranging Post-Test} > p^* \text{ Equation} \\ & \frac{\text{SSLR}_i p}{(\text{SSLR}_i p) + (1-p)} > \frac{\Delta O_{D_c}}{\Delta O_{D_c} + \Delta O_{D_t}} \\ & \text{1. Multiply through by the denominators:} \\ & (\text{SSLR}_i p \Delta O_{D_c}) + (\text{SSLR}_i p \Delta O_{D_c}) > (\text{SSLR}_i p \Delta O_{D_c}) + ((1-p) \Delta O_{D_c}) \\ & 2. \text{ Cancel } (\text{SSLR}_i p \Delta O_{D_c}) : \\ & (\text{SSLR}_i p \Delta O_{D_c}) > (1-p) \Delta O_{D_c} \\ & 3. \text{ Divide through by } p\Delta O_{D_t} \\ & \text{SSLR}_i > \frac{(1-p) \Delta O_{D_c}}{p \Delta O_{D_t}} \end{aligned}$$

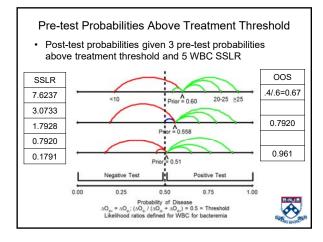
· Classification of stratum as negative test implies:

$$SSLR_i < \frac{(1-p) \Delta O_{D-}}{p \Delta O_{D-}}$$

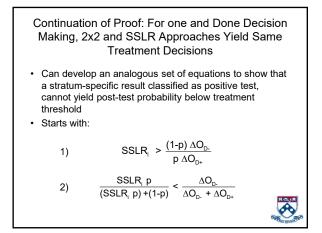
• Post-test probability above treatment threshold implies: SSLR, >  $\frac{(1-p) \Delta O_{D-}}{-}$ 

$$SLR_i > \frac{(1 p) \Delta O_D}{p \Delta O_{D+}}$$

 Thus, impossible for stratum 1) to be classified as negative test 2) if post-test probability resulting from stratum specific likelihood ratio is above treatment threshold









- In optimal 2x2 table, strata that yield post-test probabilities of disease above treatment threshold classified as positive tests
- Strata that yield post-test probabilities below treatment threshold classified as negative tests
- Thus, in One and Done decision making -- in which our treatment decision is based on whether post-test probability is >p\*/<p\* -- use of optimal 2x2 table and SSLR yield identical treatment decisions



### Relative Merits, Continuous Updating Decision Making (1)

- 1) Use of SSLRs generally yields different post-test probabilities than use of optimal 2x2 table
- 2) Use of SSLRs generally yields more DISCRIMINATING post-test probabilities than optimal 2x2
  - Why more discriminating?
    - Because SSLR with no averaging of strata yield more very high and very low post-test probabilities, whereas 2x2 approach – with more averaging across strata – have more post-test probabilities clustered around pre-test probability



# Relative Merits, Continuous Updating Decision Making (2)

 Unlike the 2x2 approach, in SSLR approach, no stratum-specific result that yields a post-test probability greater than pre-test probability will ever yield one that is less than pre-test probability (and vice versa)



IL-6 Level	W / Bact	W/O Bac
<u>&gt;</u> 10 <sup>3</sup>	4	2
<u>&gt;</u> 10² - <10³	13	9
<10 <sup>2</sup>	5	35
Total	22	46


3	3 IL-6 Categories, 4 IL-6 2x2 Tables							
IL-6 Level	W / Bact	W/O Bact	LR+	LR-				
All negative	0 22	0 46		1.000				
<u>&gt;</u> 10 <sup>3</sup>	4 18	2 44	4.182	0.855				
<u>≥</u> 10 <sup>2</sup>	17 5	11 35	3.231	0.299				
All positive	22 0	46 0	1.000					



IL-6 Level	W / Bact	W/O Bact	SSLR
<u>&gt;</u> 10 <sup>3</sup>	4	2	4.182
<u>≥</u> 10² - <10³	13	9	3.020
<10 <sup>2</sup>	5	35	0.299
Total	22	46	



	Actual Test Result				
2x2 Cut-off	<u>&gt;</u> 10 <sup>3</sup>	<u>&gt;</u> 10²; <10³	<10 <sup>2</sup>		
All negative	1.0	1.0	1.0		
<u>&gt;</u> 10 <sup>3</sup>	4.182	0.855	0.855		
<u>&gt;</u> 10 <sup>2</sup>	3.231	3.231	0.299		
All positive	1.0	1.0	1.0		
SSLR Approach	4.182	3.020	0.299		

	1	Actual Test Resul	t
2x2 Cut-off	<u>&gt;</u> 10 <sup>3</sup>	<u>&gt;</u> 10 <sup>2</sup> ; <10 <sup>3</sup>	<10 <sup>2</sup>
All negative	1.0	1.0	1.0
<u>&gt;</u> 10 <sup>3</sup>	4.182	0.855	0.855
<u>&gt;</u> 10 <sup>2</sup>	3.231	3.231	0.299
All positive	1.0	1.0	1.0
SSLR Approach	4.182	3.020	0.299

a) E10-E10 and GOEIN agree about threaten of probability shift
 2) But 2x2 approach yields too small an increase for ≥10<sup>3</sup> (3.231 vs 4.182) and too large an increase in probability for 10<sup>2</sup>-10<sup>3</sup> (3.231 vs 3.020)

Actual Test Result					
x2 Cut-off	<u>&gt;</u> 10 <sup>3</sup>	<u>&gt;</u> 10²; <10³	<10 <sup>2</sup>		
All negative	1.0	1.0	1.0		
<u>&gt;</u> 10 <sup>3</sup>	4.182	0.855	0.855		
<u>&gt;</u> 10 <sup>2</sup>	3.231	3.231	0.299		
All positive	1.0	1.0	1.0		
SLR Approach	4.182	3.020	0.299		



# Any Role Left for 2x2 approach?

- Can use SSLR to calculate post-test probabilities that are at least as "good", if not better (i.e., more discriminating), than those derived from optimal 2x2 table
- Can use either approach to identify "positive" tests / "positive" strata
- Does that mean that Jaeschke, et al. were correct that once you understand SSLR, there is no more role for selection of optimal 2x2 table?



### Maybe Not, But...

- Already have seen that SSLR throw away information that aides in the choice between tests
- While many have pointed to SSLR's more effective combination of multiple tests, i.e., continuous updating of probabilities), is continuous updating as unproblematic as the "evidence-based" commentators suggest?



### Continuous Updating and Multiple Tests

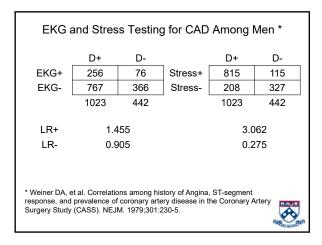
- Standard model of continuous updating assumes starting with a prior; obtaining test result; updating probability; obtaining another test result; updating;....
  - If want to use a single set of LR for each test whether or not predictions have already been made with other tests, requires that results from multiple tests be "independent"



### **Timing of Multiple Tests**

- · Multiple tests can be performed:
  - In parallel (multiple tests performed at same time)
  - In sequence
- All else equal, in parallel/in sequence choice affects cost, but not predicted probability
  - In parallel commits us to performing all tests
  - For "in sequence," if 1) test results are "independent",
     2) we use SSLR to update probabilities, same test results will yield same post-test probabilities
    - But may be sufficient information from first few tests so later tests can be avoided







# Continuous Updating: CAD Risk

- Identify pre-test probability (e.g., 20%)
- Order EKG, result is positive, post-test probability:
   (0.2\*1.455) / ((.2\*1.455)+0.8) = 0.2667
- Order exercise stress test, result is positive

   (0.2667\*3.062) / ((0.2667\*3.062)+0.7333) = 0.5269
- Would also obtain 0.5269 if we ordered stress test first - (0.2\*3.062) / ((0.2\*3.062)+0.8) = 0.4336
- Ordered EKG second
  - (0.4336\*1.455) / ((0.4336\*1.455)+0.5664) = 0.5269
- Would also obtain 0.5269 if we multiplied both LR
  - $(0.2^{*}1.455^{*}3.062) / ((0.2^{*}1.455^{*}3.062) + 0.8) = 0.5269$
  - Multiplication of LR considered an advantage of SSLR

Why Not Consider 2 Tests as Joint Test?

- 4 potential joint test results (strata)
  - ST+/EK+, ST+/EK-, ST-/EK+, and ST-/EK-
- As we know, 4 strata, 5 2x2 tables
  - All negative
  - ST+/EK+ positive
  - ST+/EK+ and ST+/EK- positive
  - ST+/EK+ and ST+/EK- and ST-/EK+ positive
  - All positive
- If tests independent, can use multiplication to calculate LR+/LR- for the five potential combinations of positive and negative test results



Can Also Calculate SSLR for 4 Combinations of 2 test's results



Effective SSL	.R, 2 Indep	pende	ent Dichotor	nousTests
ST/EK result	EKG LR		Stress LR	SSLR
ST+/EK+	1.455	Х	3.062	4.455
ST+/EK-	0.905	Х	3.062	2.771
ST-/EK+	1.455	Х	0.275	0.400
ST-/EK-	0.905	Х	0.275	0.249

### · Results ordered by descending SSLR

 Comparison of OOS to resulting "joint" SSLR indicates which combinations should be considered positive

 e.g., if OOS= 0.3, ST+/EK+, ST+/EK-, and ST-/EK+ should be included in a "positive" test

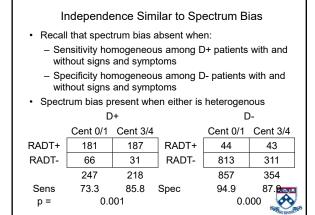
# Issues With Treating 2 Tests Jointly

- Use of combination of tests as a "joint" test returns us to one and done decision making rather than continuous updating
  - As with ordinary one and done decision making will reach same treatment decisions by use of either optimal combined 2x2 table or test's SSLR
- If want to calculate joint test characteristics using LR / Sens & Spec for each test when used separately, requires test independence
- But Are EKG and stress tests independent?



TEST INDEPENDENCE





# Similarities Between Spectrum Bias and Independence

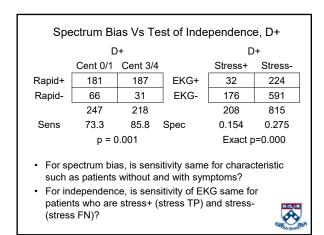
- Evaluation of spectrum bias and test independence have a number of similarities
  - For both assessments, evaluation conducted separately, one 2x2 table among those with disease and one 2x2 table among those without disease
  - Rows represent test results (T+ and T-)
  - Statistically significant differences indicate evidence for presence of spectrum bias AND for lack of independence
    - Spectrum bias / lack of independence present when either sensitivities or specificities differ significantly



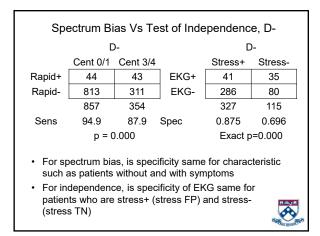
### Primary Difference

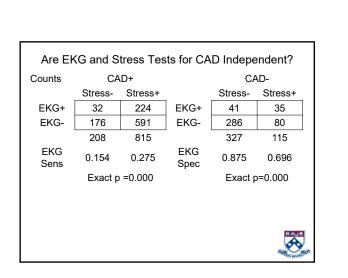
- For spectrum bias, columns represent presence or absence of characteristic for which spectrum bias is being assessed, e.g.:
  - Patients with symptoms vs those without
  - Females vs males
  - Old vs young, etc.
- For test of independence columns represent test results (T+ and T-) for second test

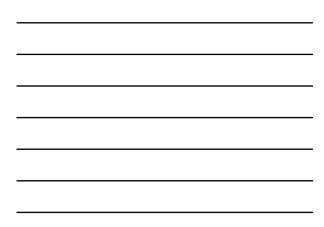


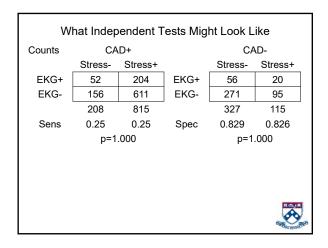




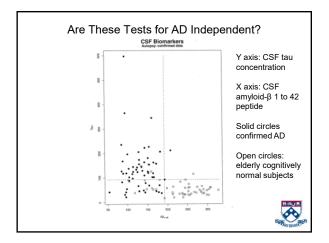














Independence of Tau and $A\beta_{1-42}$ ?						
	Alzheimer's			No Alzheimer's		
	Αβ+	Αβ-		Αβ+	Αβ-	
Tau+	37	2		3	1	
Tau-	17	0		9	39	
SE	68.5%	100%	SP	75%	97.5%	
	p = 1.0 (exact) *			p = 0.03 (exact) †		
<ul> <li>Lack of independence due to very different proportions of Tau+ and Tau- among Aβ+ (3/12) and Aβ- patients (1/40)</li> </ul>						
* tabi 37 2 \ † tabi 3 1 \						



### Agreement and Independence

- Is strong agreement between 2 tests suggestive of independence or lack of independence?
  - By strong agreement, we mean:
    - i.e., When one test is positive other is almost always positive
    - When one is negative other is almost always negative



### Strong Agreement...

- ...suggestive of LACK of independence
  - Independence holds if, for example, among diseased individuals fraction of test 1's true positives equal among test 2's true positives and false negatives
     But if tests demonstrate strong agreement:
  - When test 2's results are positive, test 1's results also likely to be positive
    - i.e., too few positives among test 1's false negatives
  - When test 2's results are negative, test 1's results also likely to be negative
    - i.e., too few negatives among test 1's false positives



#### Informative Lack of Agreement

- Independence implies "informative lack of agreement"
  - Want impact of false negative result from 1 test to be offset by true positive from second
  - Want impact of false positive result from 1 test to be offset by true negative from second
- Strong agreement implies both tests will generally provide same misinformation
  - i.e., both false negative or both false positive



### Implication of Dependence for Continuous Updating

- Lack of independence undermines chaining both 2x2 and SSLR approaches for updating probabilities

   i.e., Can't multiply EKG's LR- times either Stress LR
- One response to test dependence is to treat 2 dependent tests as a single test
- Again, four potential joint test results
   ST+/EK+, ST+/EK-, ST-/EK+, and ST-/EK-
- Unlike independent tests, don't calculate combined SSLR / LR+/LR- by multiplying each test's SSLR / LR+/LR-
- Instead return to original data and calculate combined SSLR / LR+/LR- from combined results of 2 tests



	Observed (Dependent) Vs Calculated (Independent) SSLR, Combined Tests *				
ST/EK result	D+	D-	Dep SSLR	Indep SSLR	P-value †
ST+/EK+	224	35	2.765	4.455	0.0004
ST+/EK-	591	80	3.192	2.771	0.02
ST-/EK+	32	41	0.337	0.400	0.33
ST-/EK-	176	286	0.266	0.249	0.04
Total	1023	442			
See data from slide 47     P-values derived via bootstrap					



### Continuous Updating and Test Dependence (2)

- As with independent tests, treating 2 dependent tests as a single joint test reduces what looked like continuous updating to one-and-done decision making
  - Selection of optimal joint test 2x2 table yields same treatment decision as SSLR
  - Purported advantage of SSLR may disappear



# SSLR = 1.0

- Using SSLR, a test can be useful even if some test results yield post-test probabilities equal to our pre-test probabilities (i.e., if one stratum has an SSLR of 1.0)
- In 2x2 approach, if a positive (negative) test result for a particular table yields post-test probabilities equal to our pre-test probabilities:
  - 1) Negative (positive) test result for same table does so as well
  - 2) ROC curve falls on  $45^{\circ}$  line
  - 3) Test provides no information



2x2 Approach	Stratum-Specific Approach
Combines proportions of populations having test results in different strata to develop likelihood ratios for positive and negative tests	<ul> <li>Does not average among strata (but does average within a stratum)</li> </ul>
→ For some patients, a test result yields post-test probabilities that are higher than pre-test, while for others same test result yields post- test probabilities that are lower than pre-test	→ A given test result yields post-test probability that is either always higher or always lower than pre-test probability

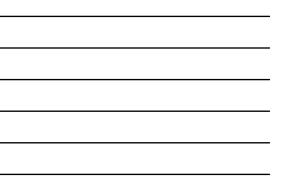
2x2 Approach	Stratum-Specific Approach
All strata whose results leave us above treatment threshold will be classified as positive and all strata that leave us below threshold will be classified as negative	<ul> <li>Strata always have same likelihood ratio (Can determin which strata are "positive" an which are "negative" by comparison to OOS)</li> </ul>
→ If only decision remaining is to treat or withhold treatment, two approaches yield same result; if other choices are available, results can differ	→ If only decision remaining is to treat or withhold treatment, two approaches yield same result; if other choices are available, results can differ

2x2 Approach	Stratum-Specific Approach
<ul> <li>Retains concept of positive test</li> </ul>	<ul> <li>Retains concept of positive test result if we compare SSLR to OOS</li> </ul>
<ul> <li>Cost of mistakes and thresholds built into definition of a positive test</li> </ul>	• Cost of mistakes and thresh- olds built into definition of a "positive" stratum
<ul> <li>Can use Bayes theorem or likelihood ratio approach to adjust pre-test probabilities</li> </ul>	<ul> <li>Can use likelihood ratio approach to adjust pre-test probabilities</li> </ul>

2x2 Approach	Stratum-Specific Approach
<ul> <li>Withhold testing for therapeutic decisions if no stratum-specific result can shift post-test probability and pre-test to opposite sides of underlying treatment threshold</li> </ul>	<ul> <li>(IF One (Test) and DONE) Withhold testing for therapeutic decisions if no stratum-specific result can shift post-test and pre-test to opposite sides of underlying treatment threshold</li> </ul>

2x2 Approach	Stratum-Specific Approach
Does not have to prespecify	<ul> <li>Does not have to prespeci</li></ul>
particular cut-points for	particular cut-points for
diagnostic test results (i.e.,	diagnostic test results (i.e.,
identify cut-off and Se and Sp	identify cut-off associated v
associated with tangency of	point on OOS that has a sl
OOS and ROC curve)	equal to OOS)

I



### Take Home Messages

- 1) We base decisions on estimates of probability of disease and one's treatment thresholds
- 2) We perform diagnostic tests to change our probabilities / certainty about appropriate treatment
- 3) We use a test's sensitivity and specificity or its likelihood ratios to revise our pre-test probability of disease to yield a post-test probability based on whether a test's results are positive or negative or based on its stratum-specific likelihood ratios
- 4) Different sensitivities and specificities may be appropriate for different patients

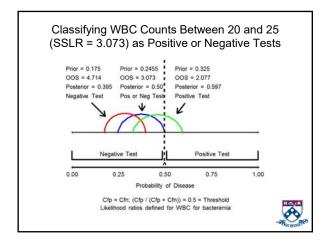


#### Take Home Messages (cont.)

- Combining data from ROC curves OR SSLR with OOS represents a good method for identifying appropriate sensitivity and specificity for a given patient
- 6) In One and Done decision making, optimal sensitivity and specificity yields a decision identical to one based on stratum-specific likelihood ratios; in continuous updating decision making, use of stratum-specific results yields superior post-test probabilities of disease



	Extra Slides	





### Do Differences Make a Difference?

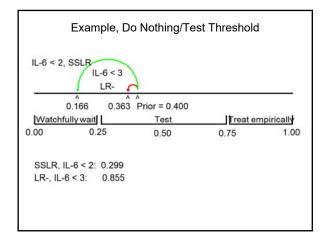
- If no other test is available or if every SSLR moves you outside threshold for additional testing
  - Two methods yield same treatment decision, because they both leave you on same side of underlying treatment threshold
  - Conclusion true even if likelihood ratios from 2x2 approach and SSLR are on opposite sides of 1.0



### Do Nothing/Test & Test/Treat Thresholds

- If tests are independent and some stratum-specific results leave you within testing range and others move you outside it, 2x2 approach will yield more mistakes
  - SSLR moves you outside testing range, but dilution from 2x2 combination of test results leaves you within testing range
    - More likely when there are extreme results yielding high SSLR and these SSLR are averaged with other smaller SSLR in 2x2 approach







# Test/Treat Thresholds

 Alternatively, SSLR may leave you within testing range (e.g., 3.020 for IL-6 between 2 and 3), but overstatement of effect of test result from LR+ (3.231 for IL-6 ≥ 2) can move you outside this range

